Noisy Rumor Spreading and Plurality Consensus

Emanuele Natale[†]

joint work with

Pierre Fraigniaud[*]

* CNRS LIAFA

† SAPIENZA UNIVERSITÀ DI ROMA

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Biological Distributed Algorithms

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Rumor-Spreading Problem

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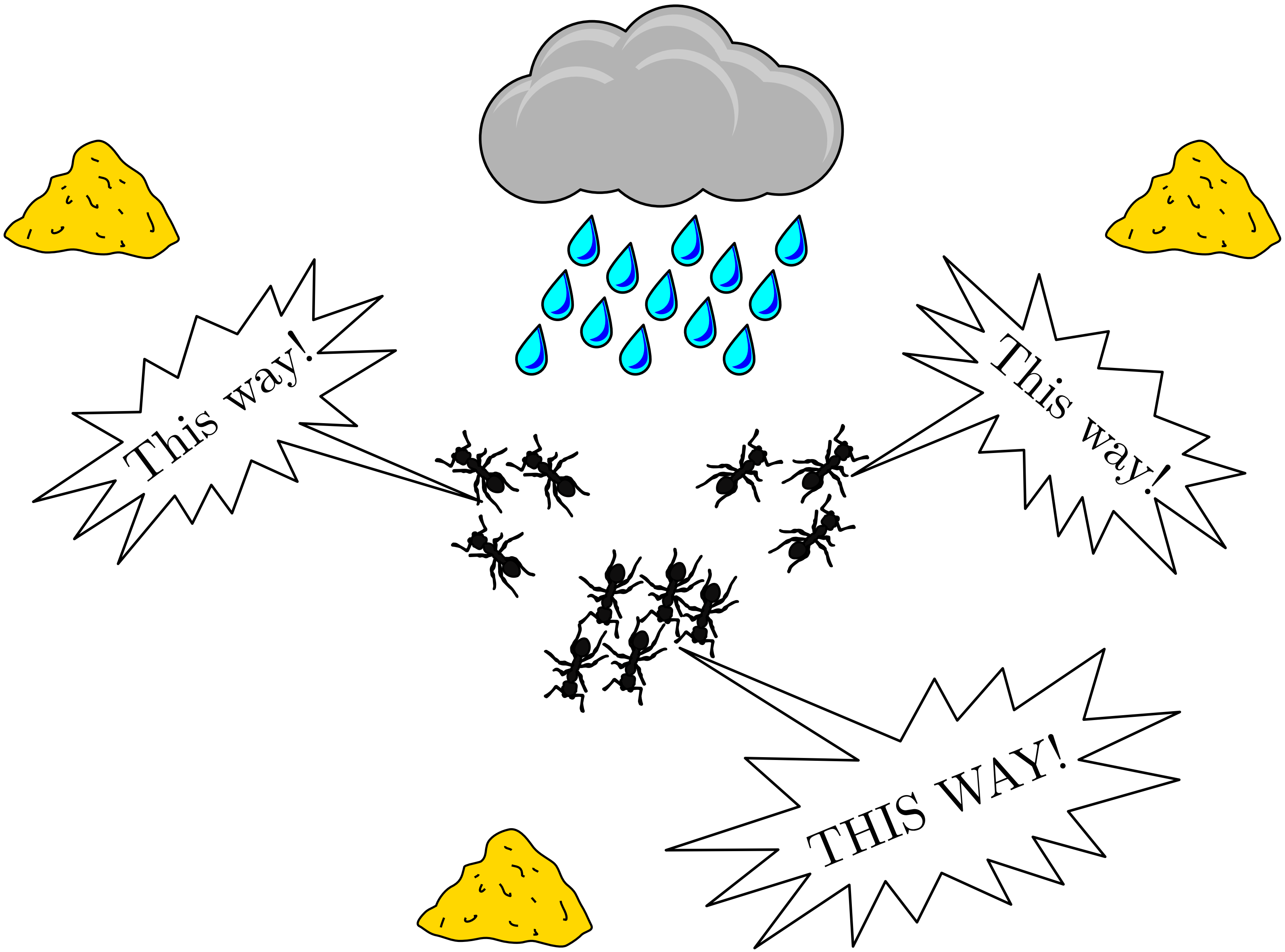
Plurality Consensus Problem



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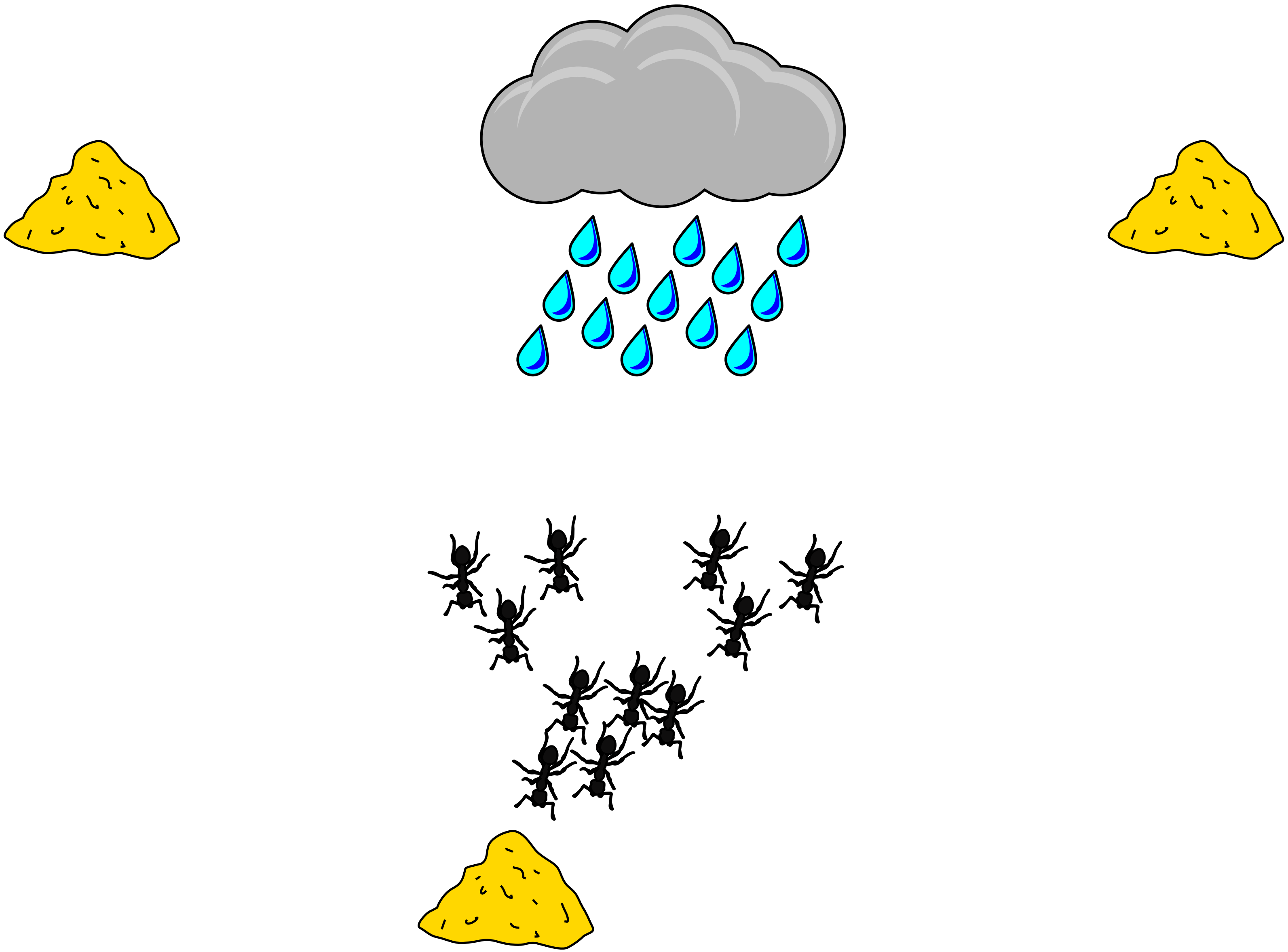
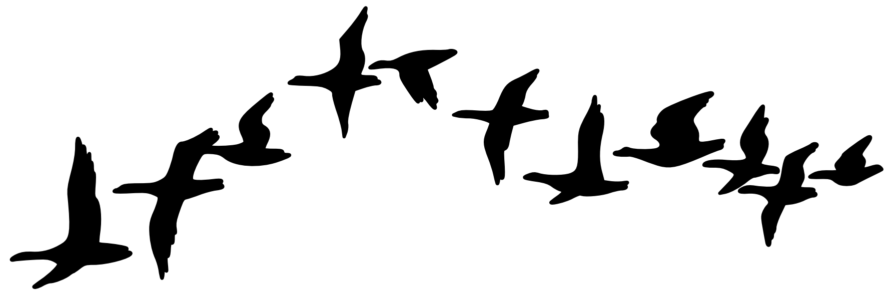


Some examples (Plurality Consensus)

Flocks of birds [Ben-Shahar et al. '10]



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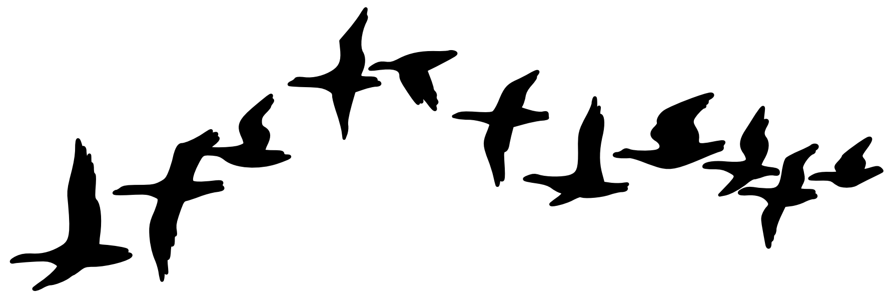
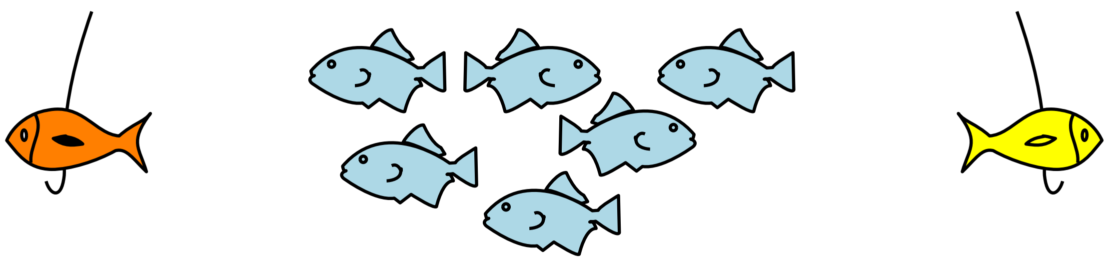


Schools of fish [Sumpter et al. '08]



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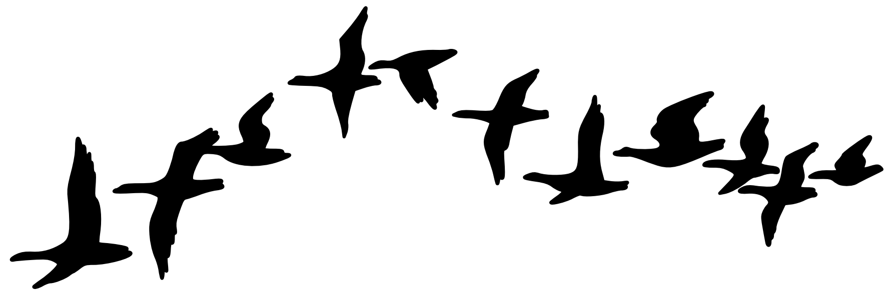

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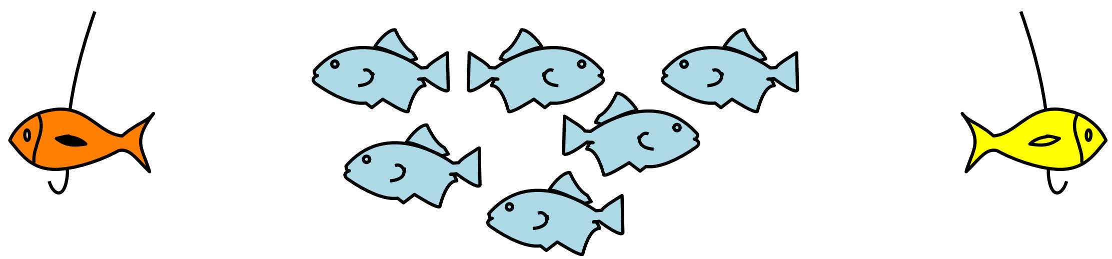
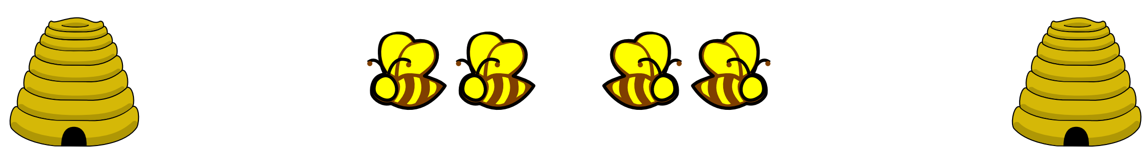


Insects colonies [Franks et al. '02]



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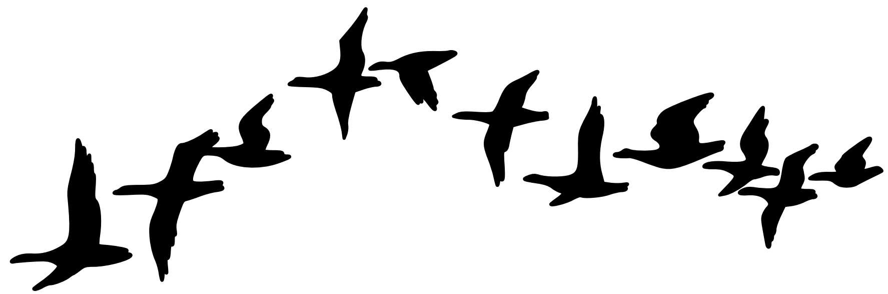

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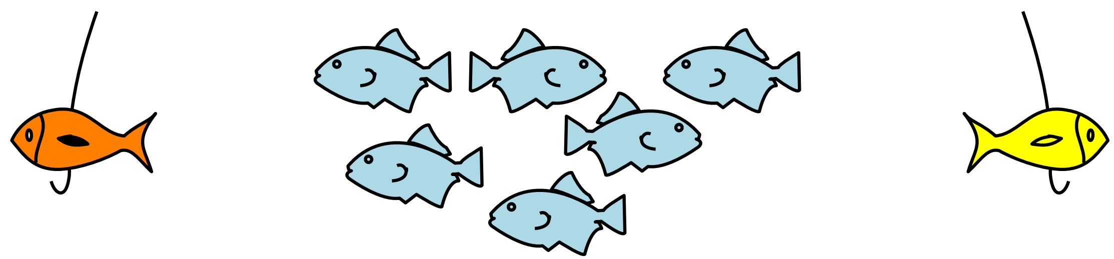

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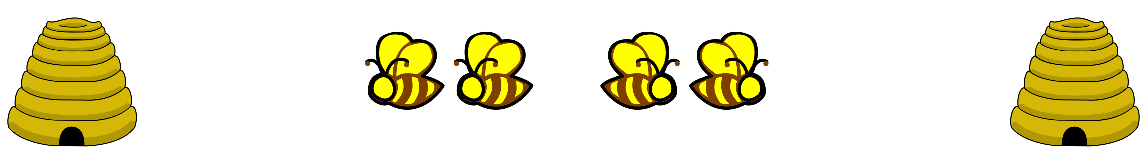

Eukaryotic cells [Cardelli et al. '12]

Animal Communication Despite Noise

Noise affects animal communication,
but animals cannot use *coding theory*...

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$\Longrightarrow$ **Natural** rules efficiently solve rumor spreading and plurality consensus despite noise.

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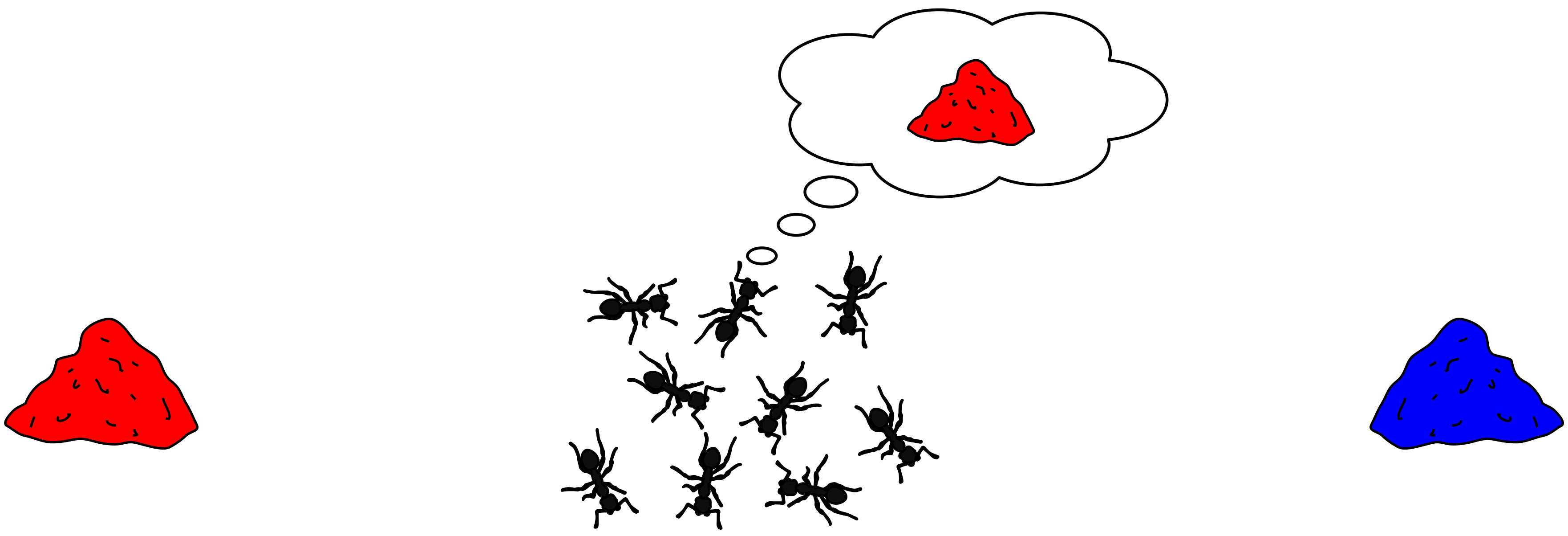
$\Longrightarrow$ **Natural** rules efficiently solve rumor spreading and plurality consensus despite noise.

They only consider the binary-opinion case.
Our contribution: generalize to many opinions.

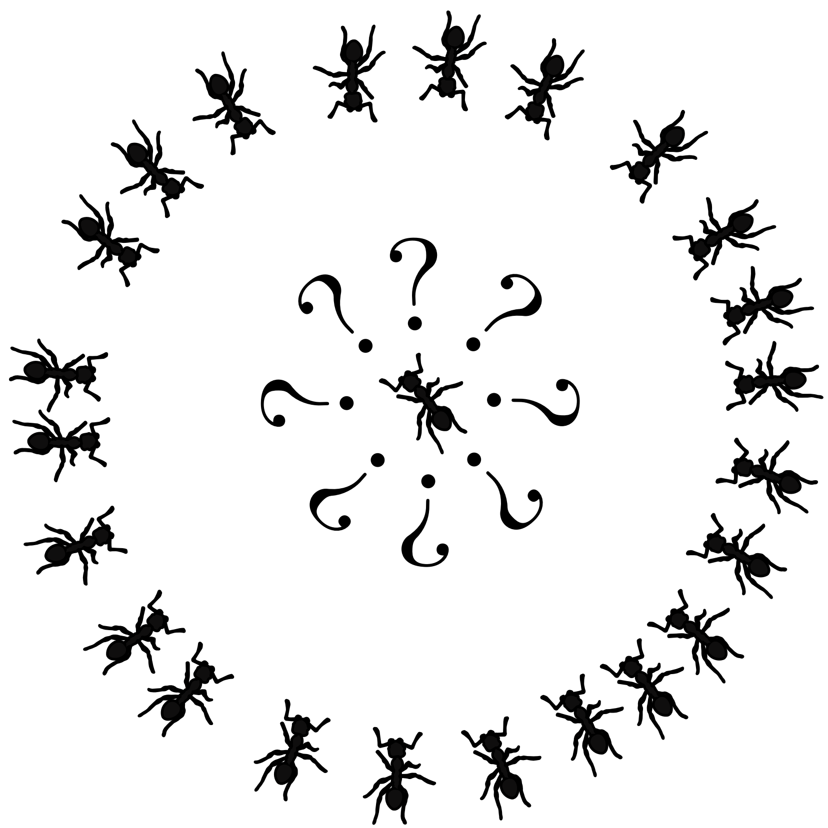
Binary Case - Model

n agents. One agent has one bit to spread.



Binary Case - Model

Communication model: push gossip model [Pittel '87]: at each round each agent can send a bit to another one chosen uniformly at random.



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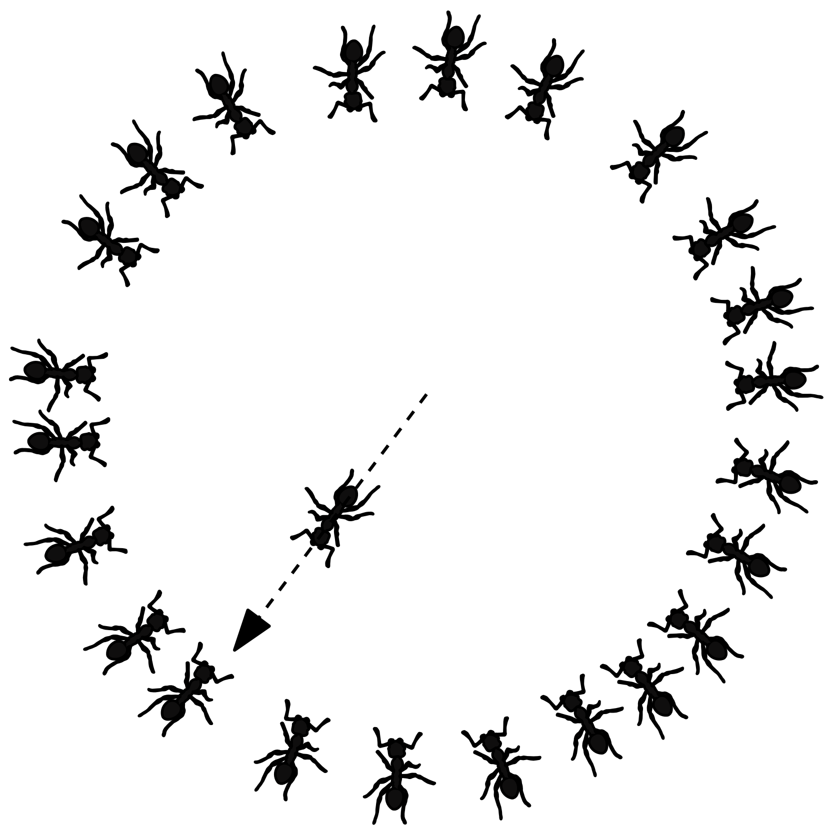
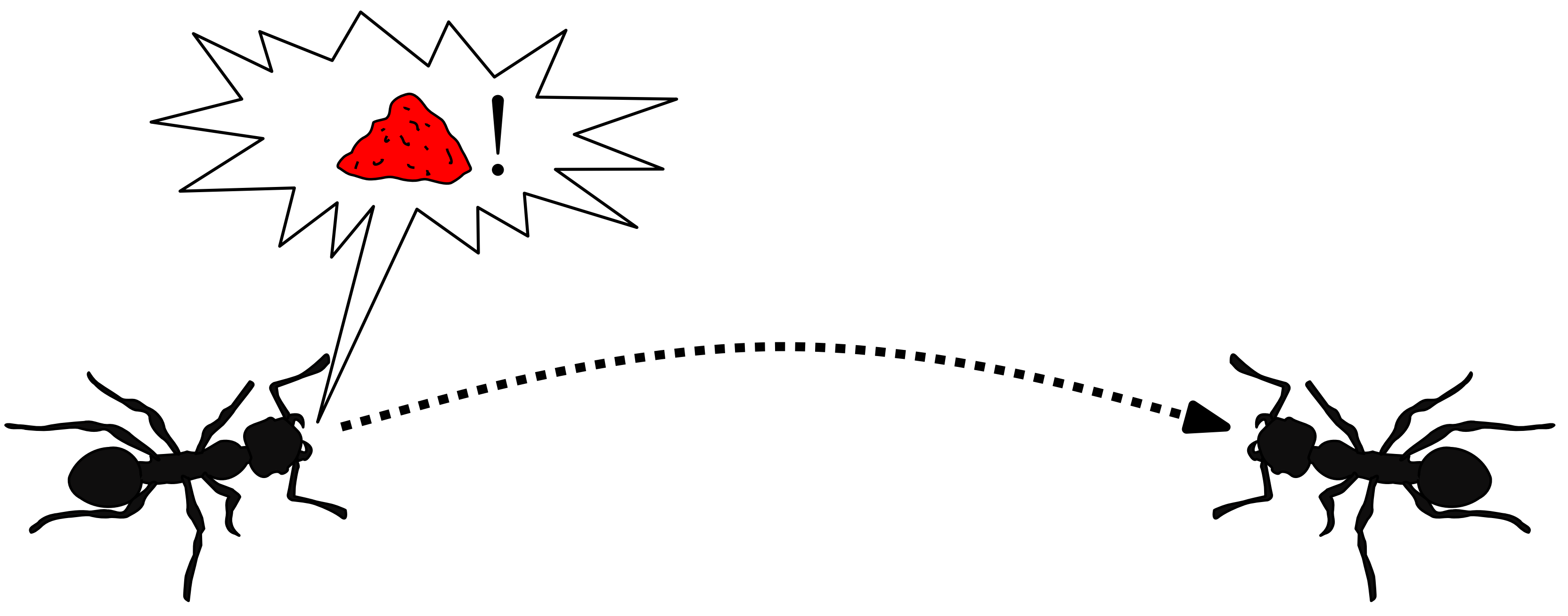


Binary Case - Model

Noise: before being received, each bit is flipped with probability $1/2 - \epsilon$ ($\epsilon = n^{-const}$).



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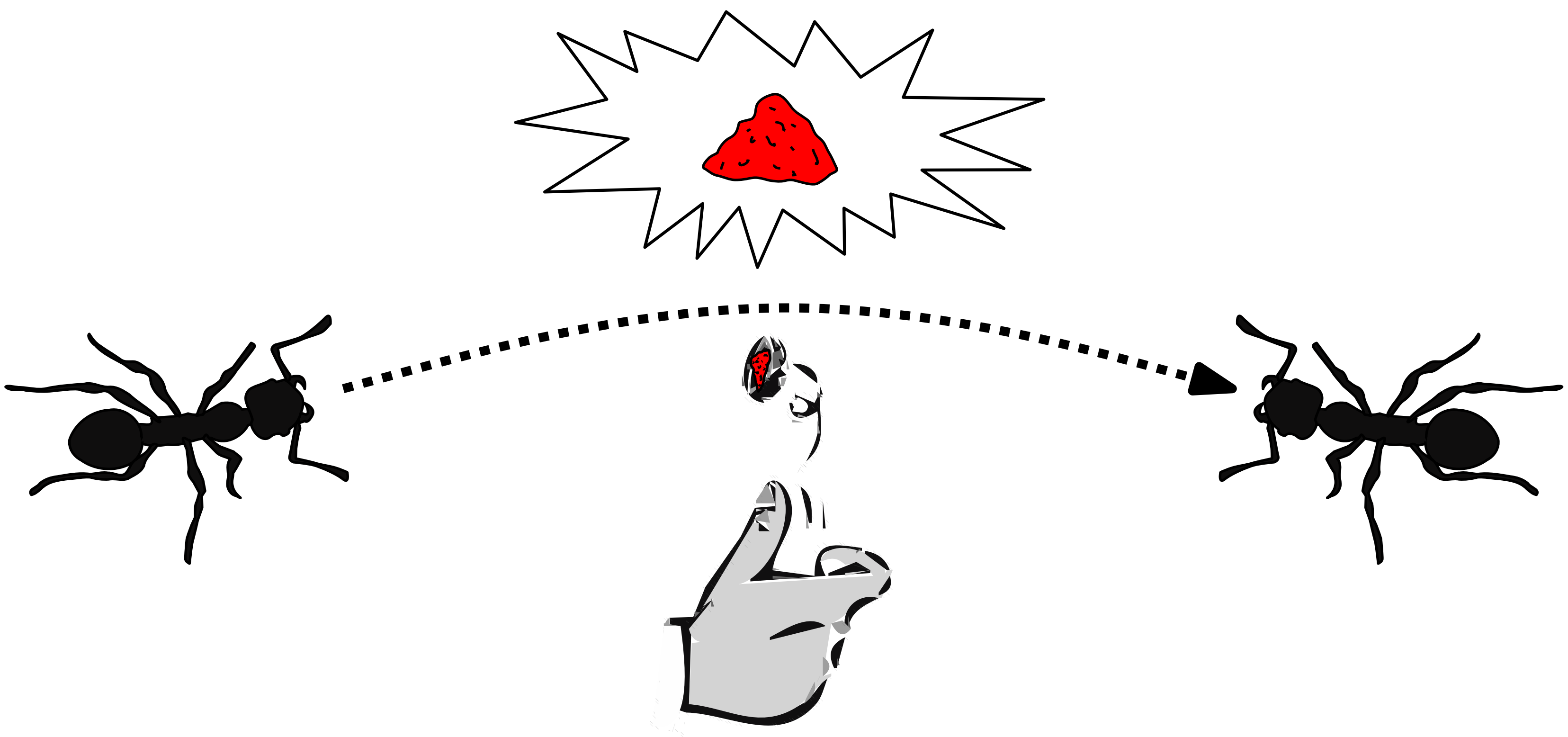

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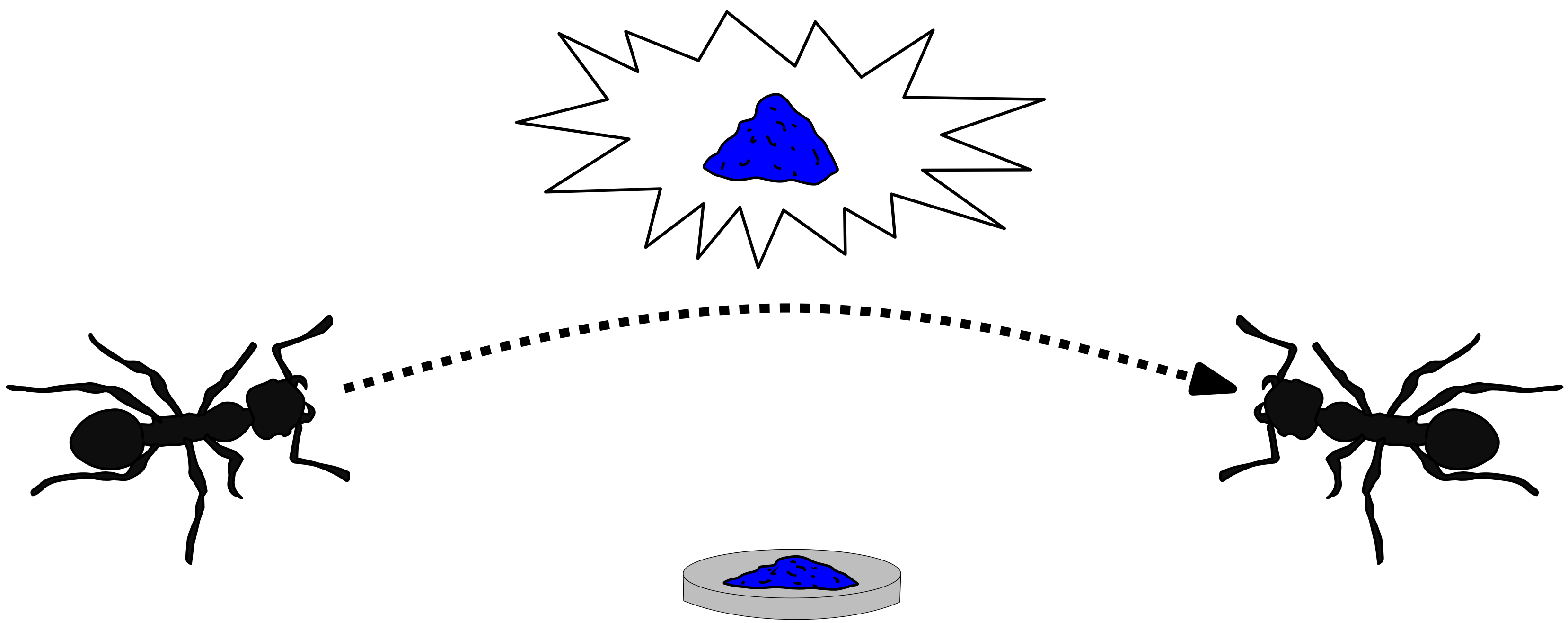

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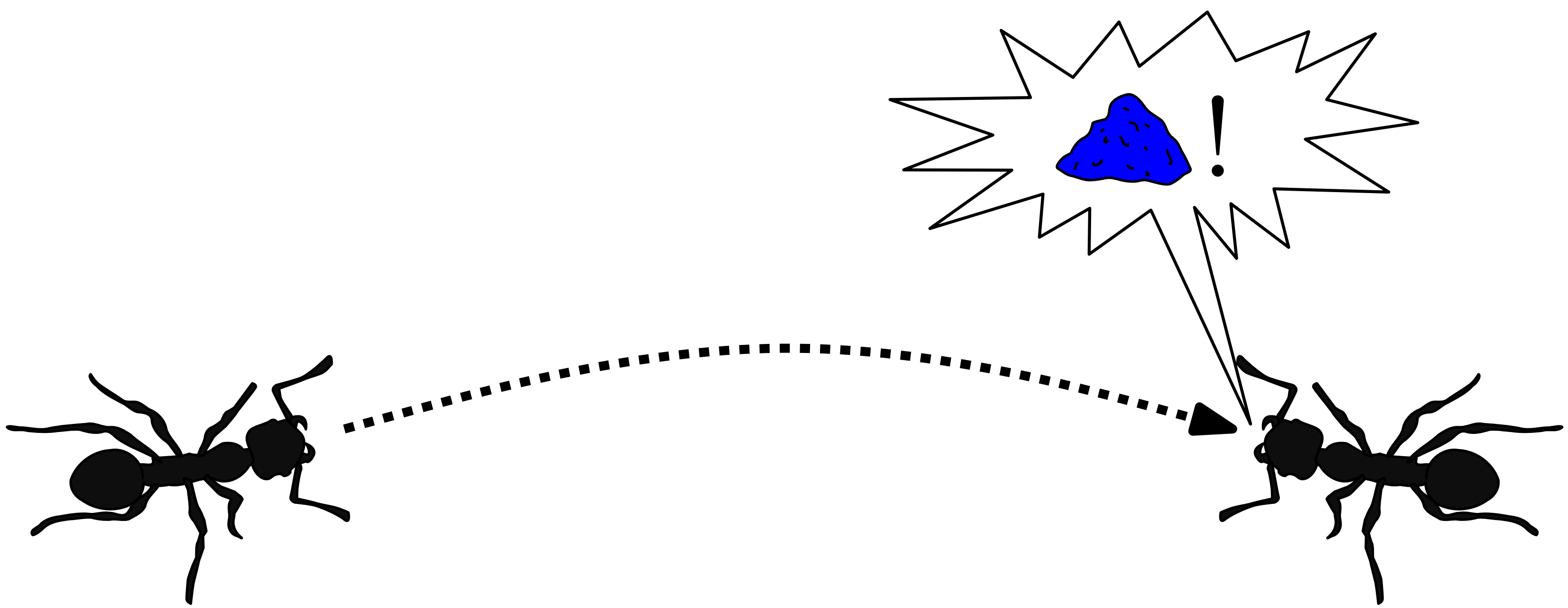
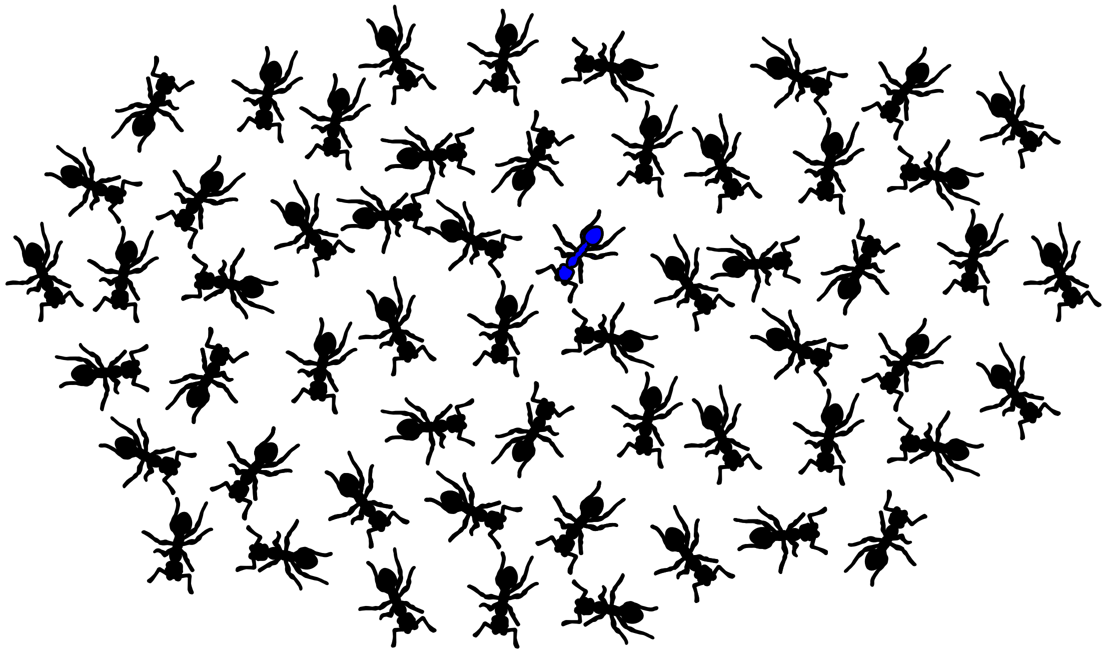


Breathe Before Speaking



trivial strategy

blue vs red:
1/0

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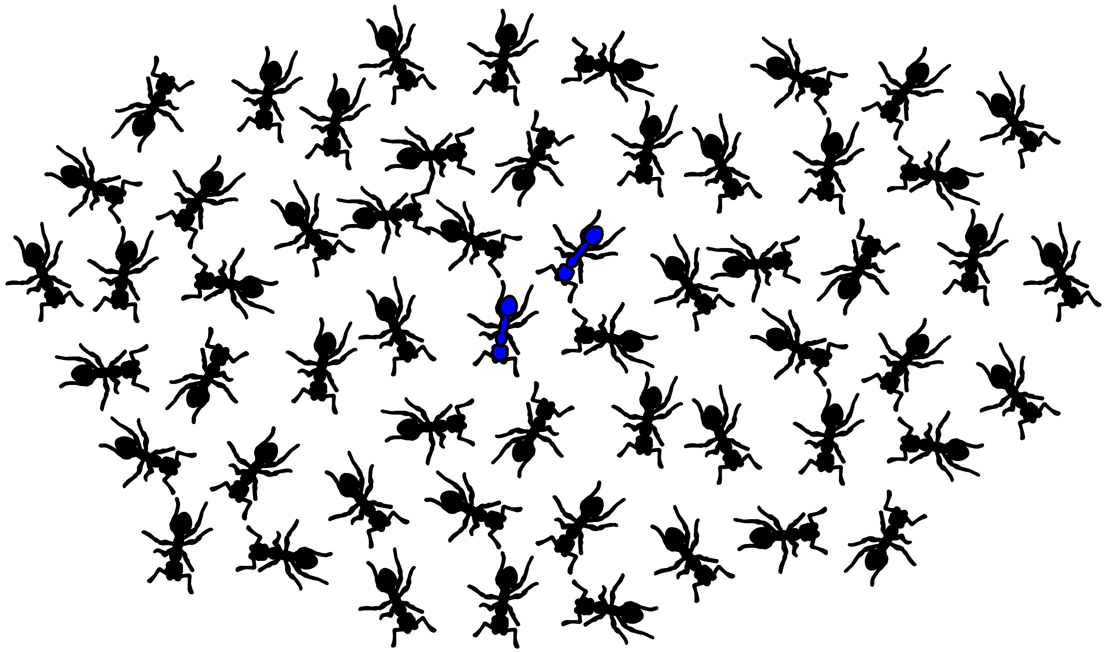

trivial strategy

blue vs red:
2/0

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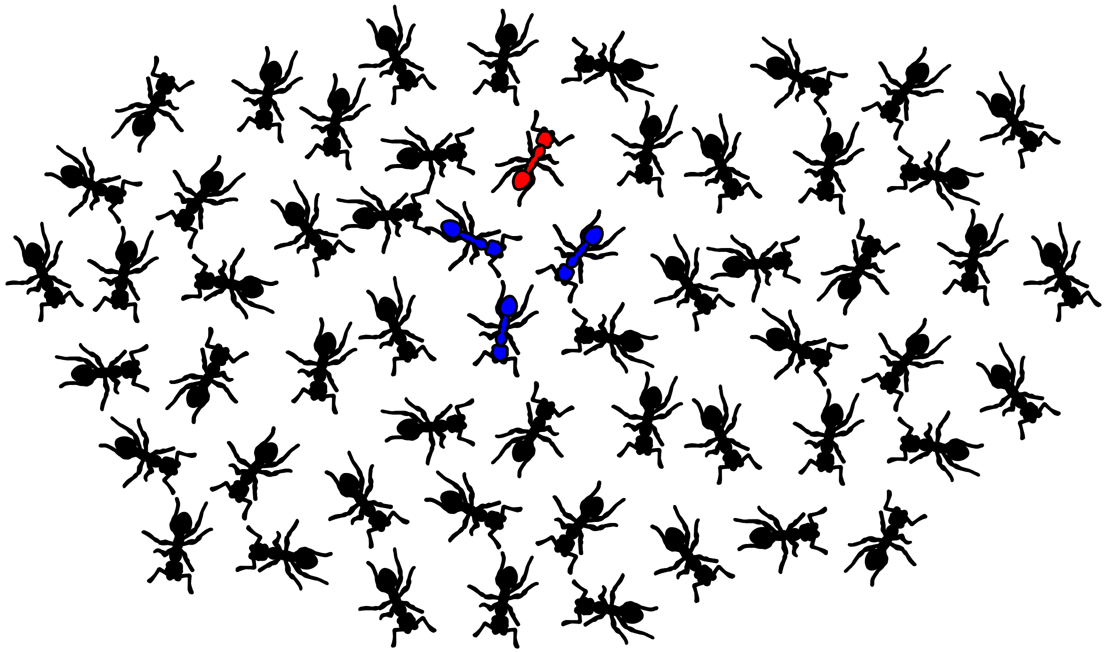

trivial strategy

blue vs red:
3/1

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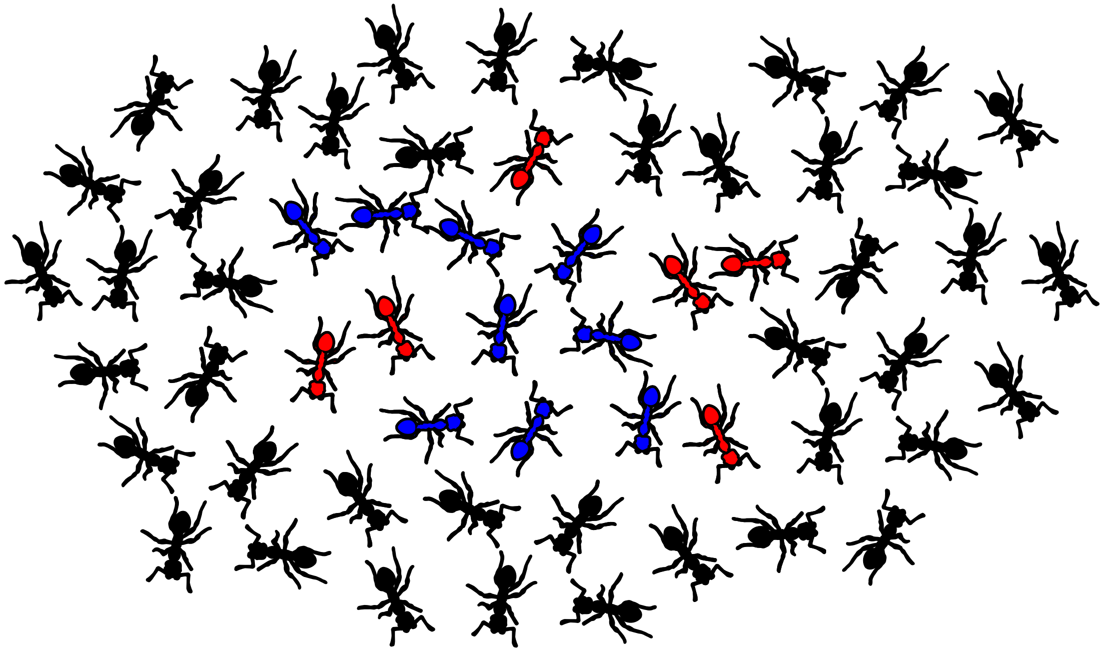

trivial strategy

blue vs red:

$9/6 = 1.5$

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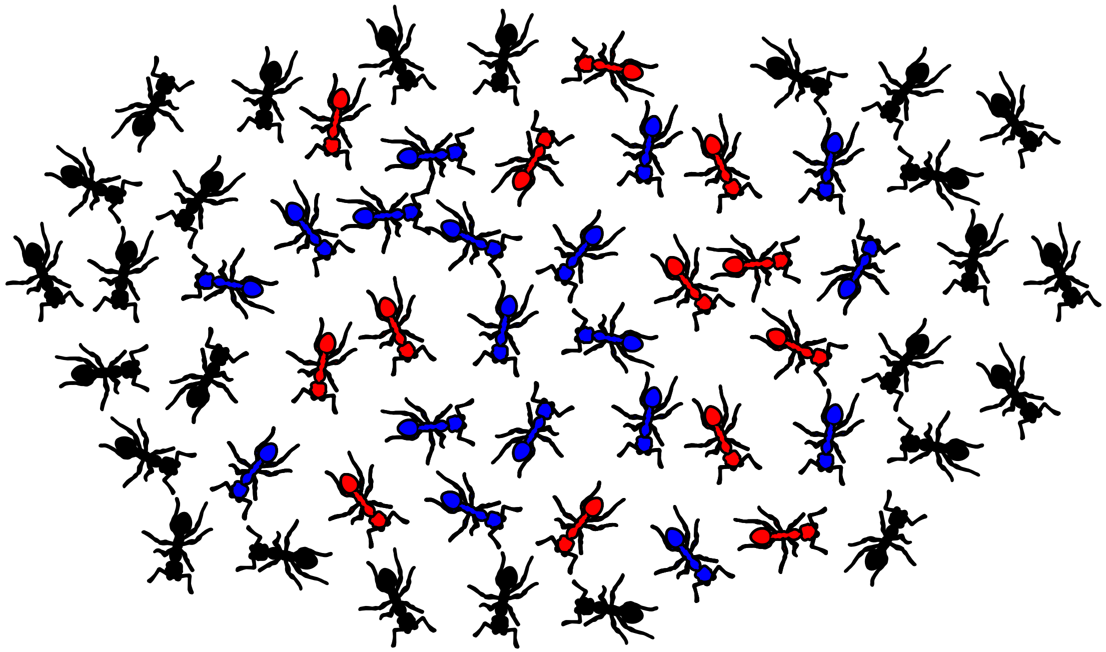
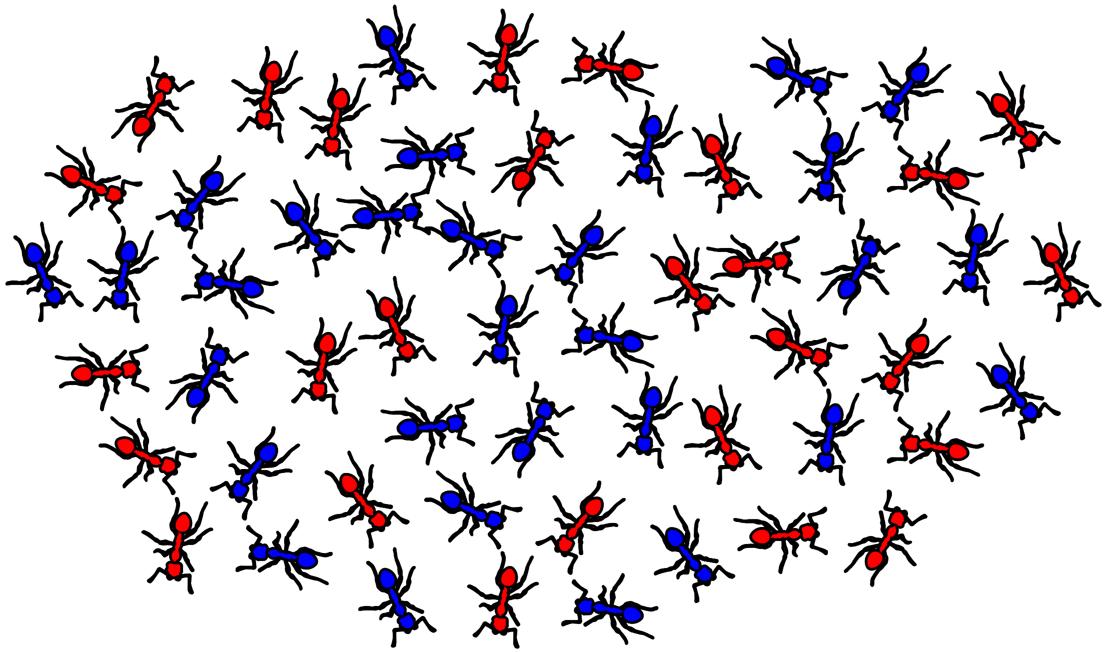


trivial
strategy

blue vs red:
18/13 $\approx$ 1.4

Breathe Before Speaking



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$35/29 \approx 1.2$

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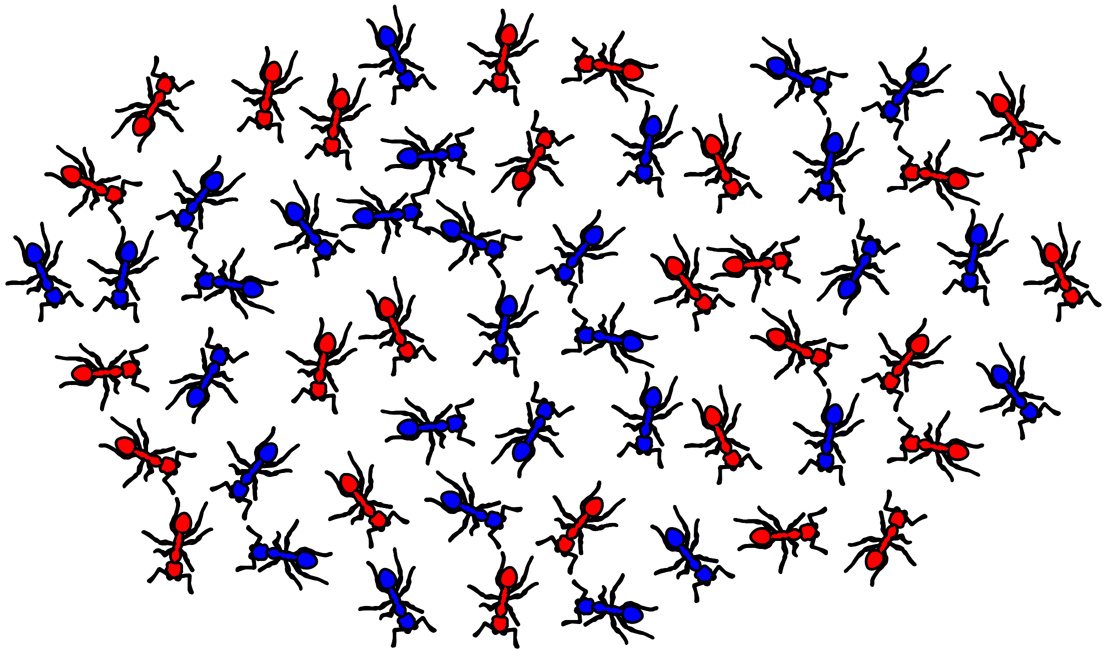

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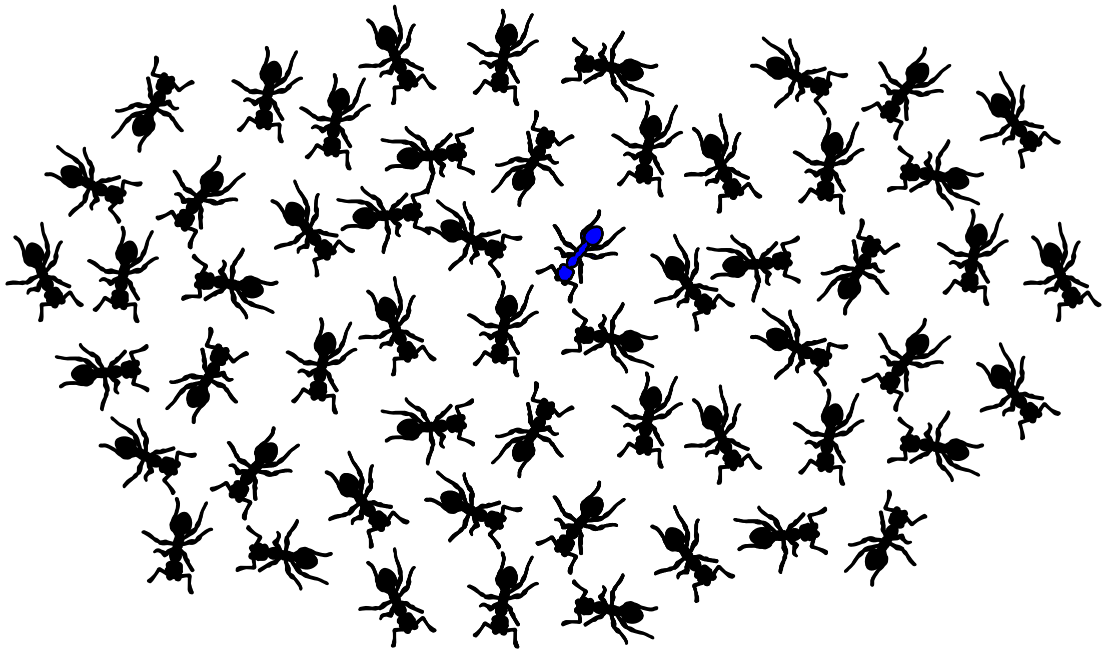

Stage 1: Spreading

blue vs red:
1/0

"[...] ants effectively self-restrict their own tendency to engage in further interactions that would excite further nest-mates."

(Razin et al. '13)

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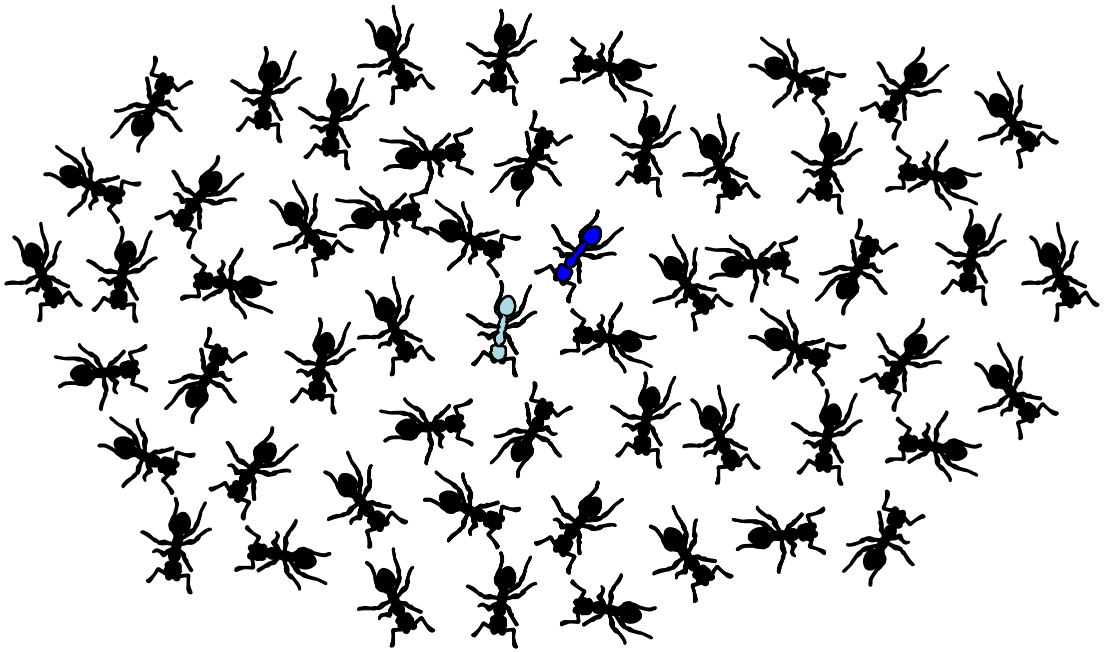

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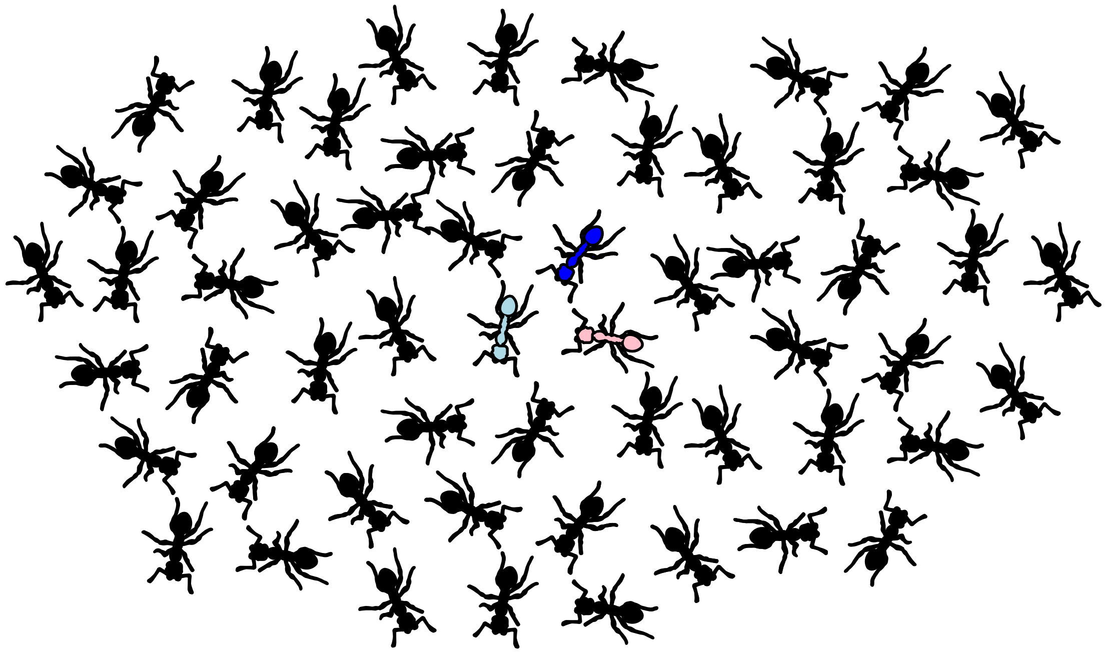

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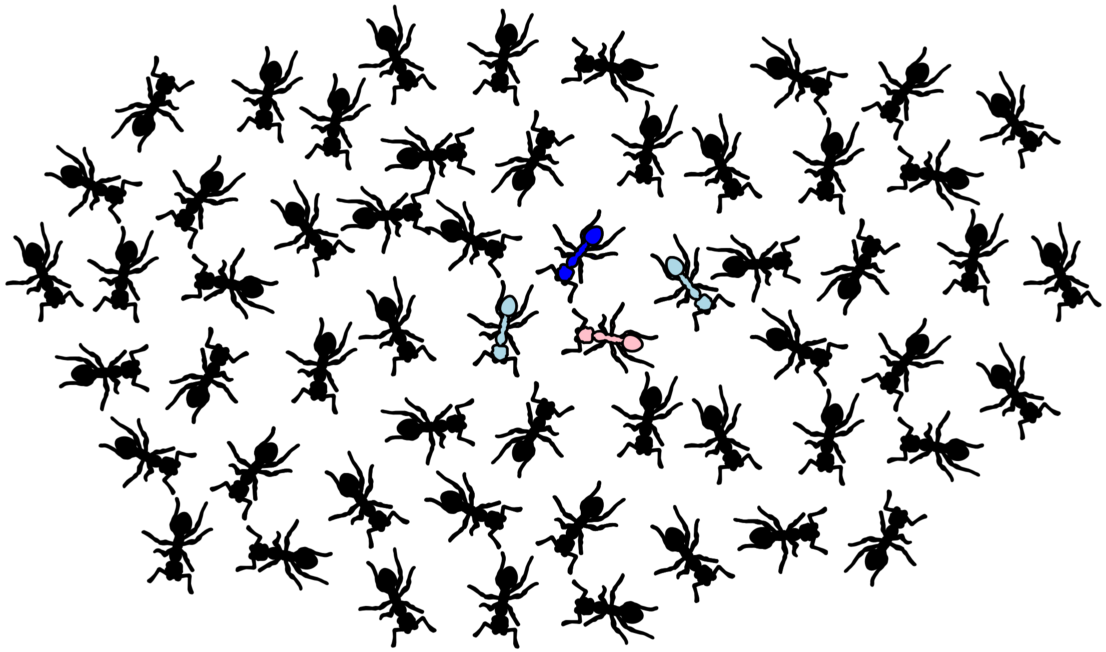

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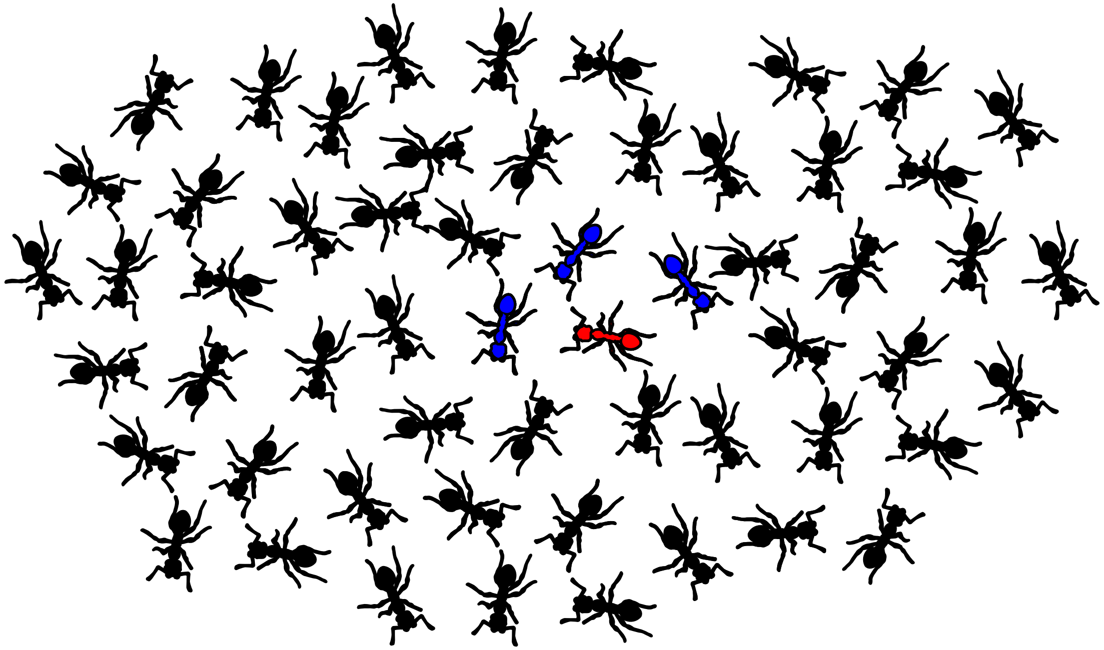

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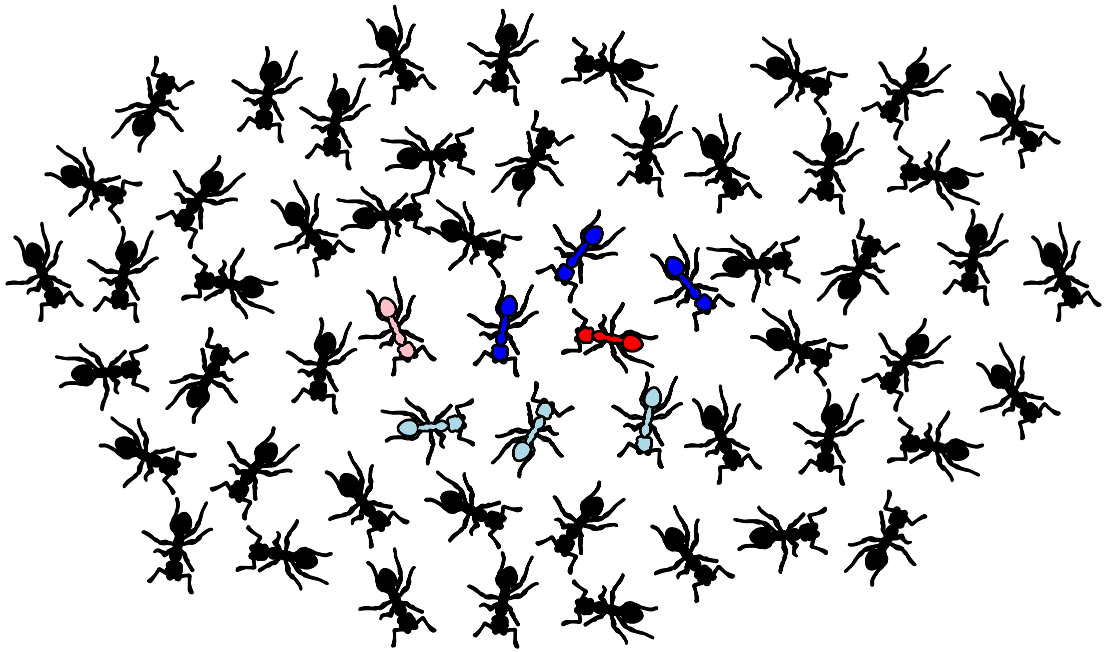

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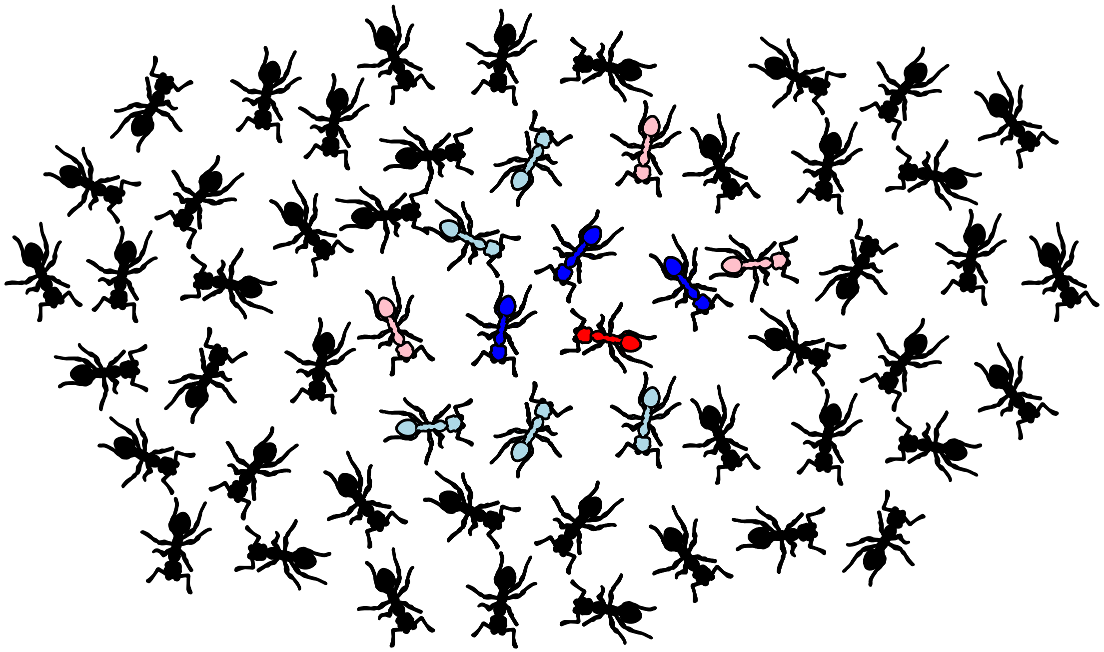

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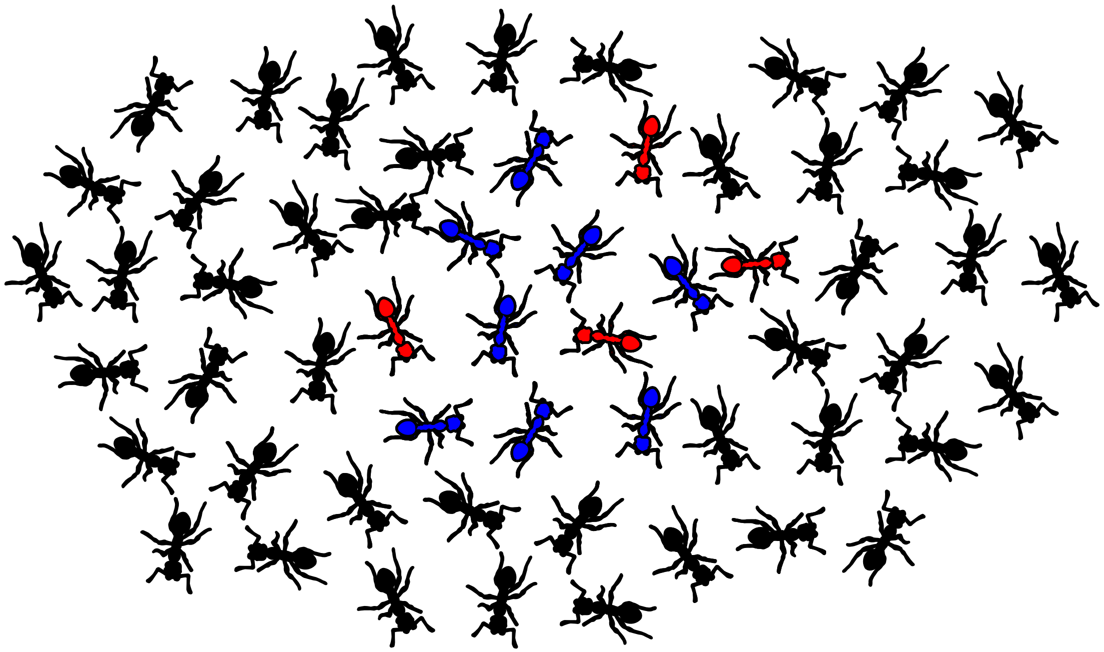

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blue vs red:
8/4

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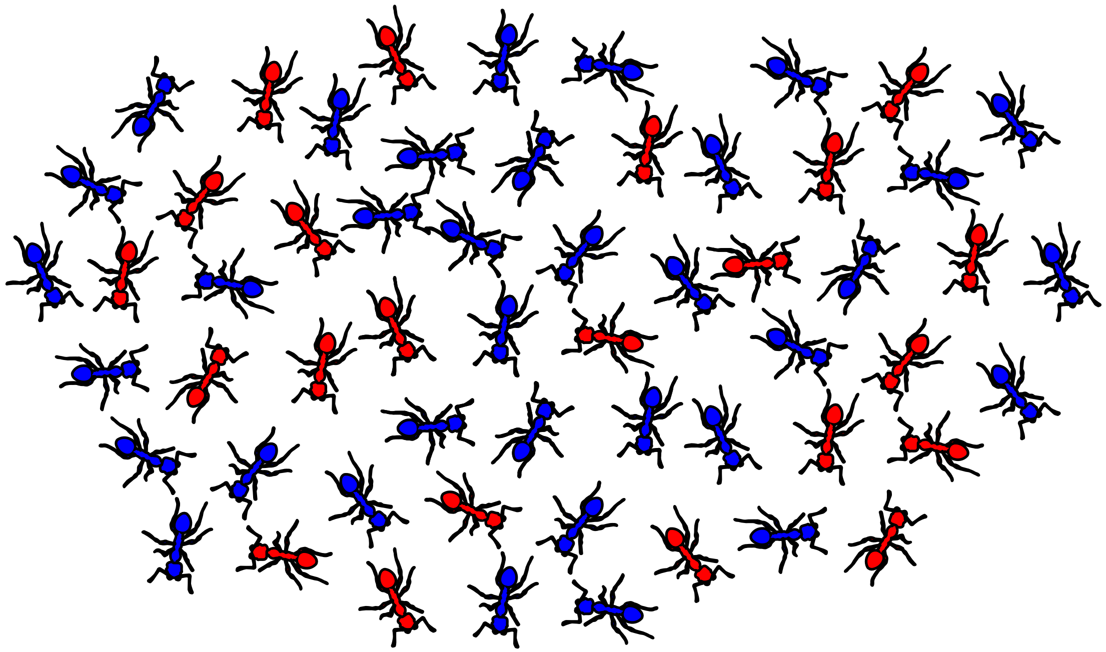

Stage 1: Spreading

blue vs red:
$40/24 \approx 1.7$

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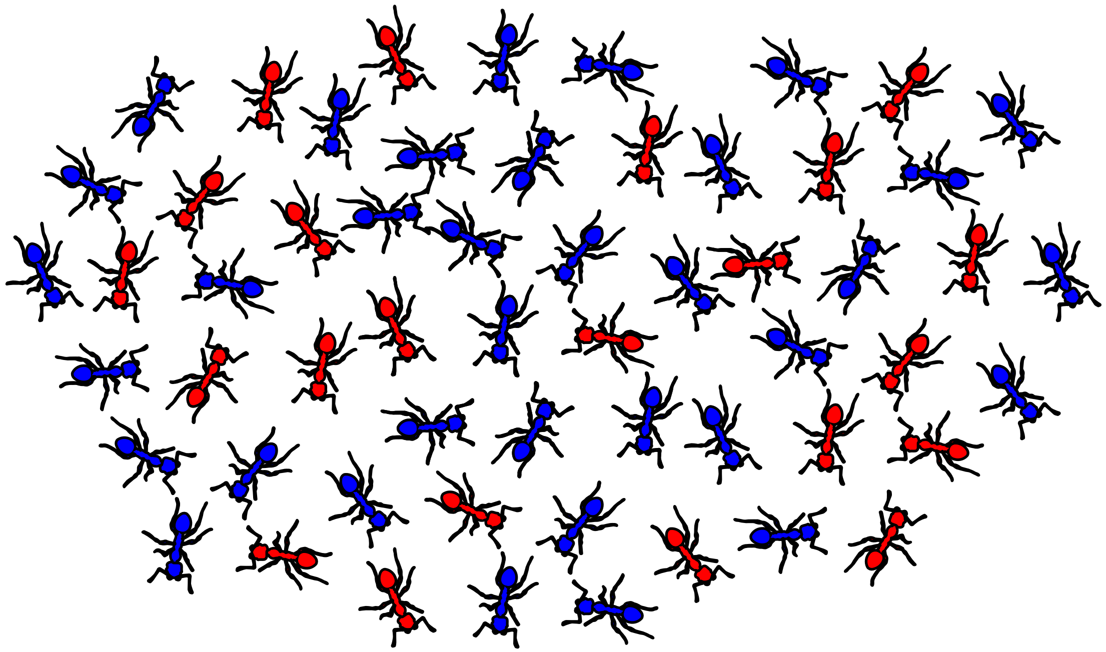

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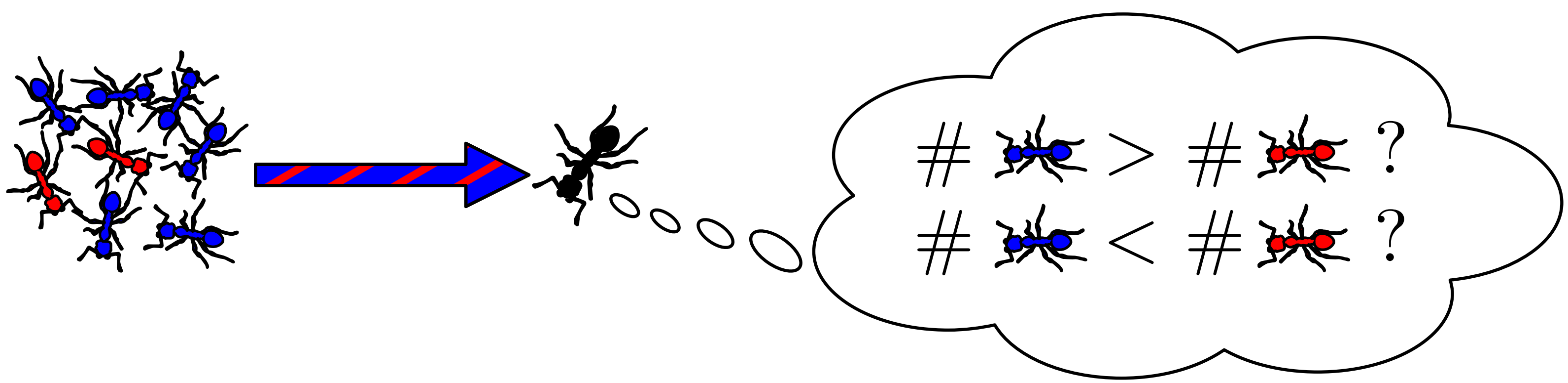
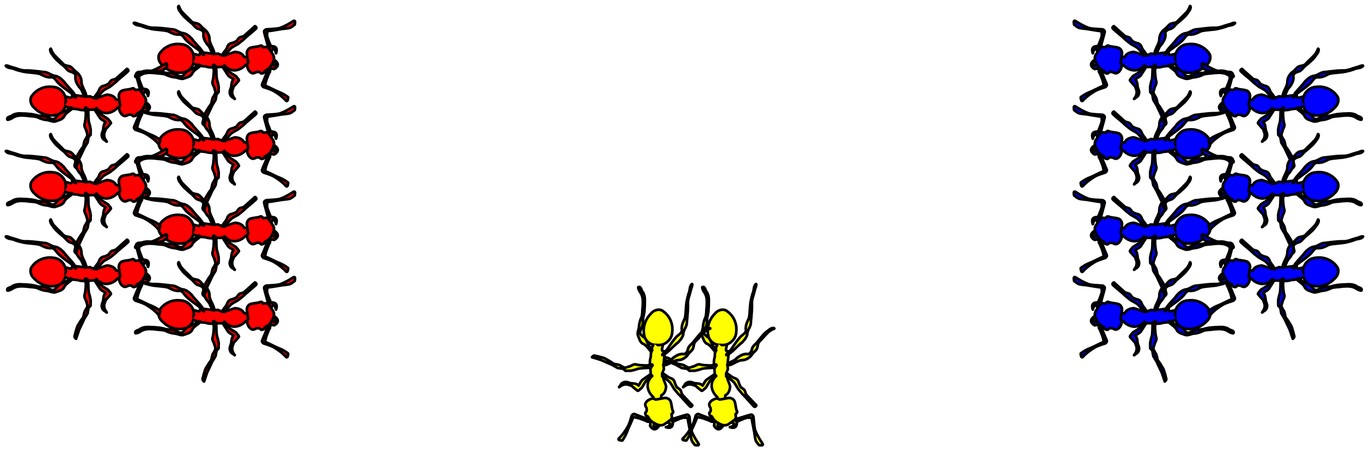
Stage 2: Amplifying majority



Mathematical Challenges

- Stochastic Dependence



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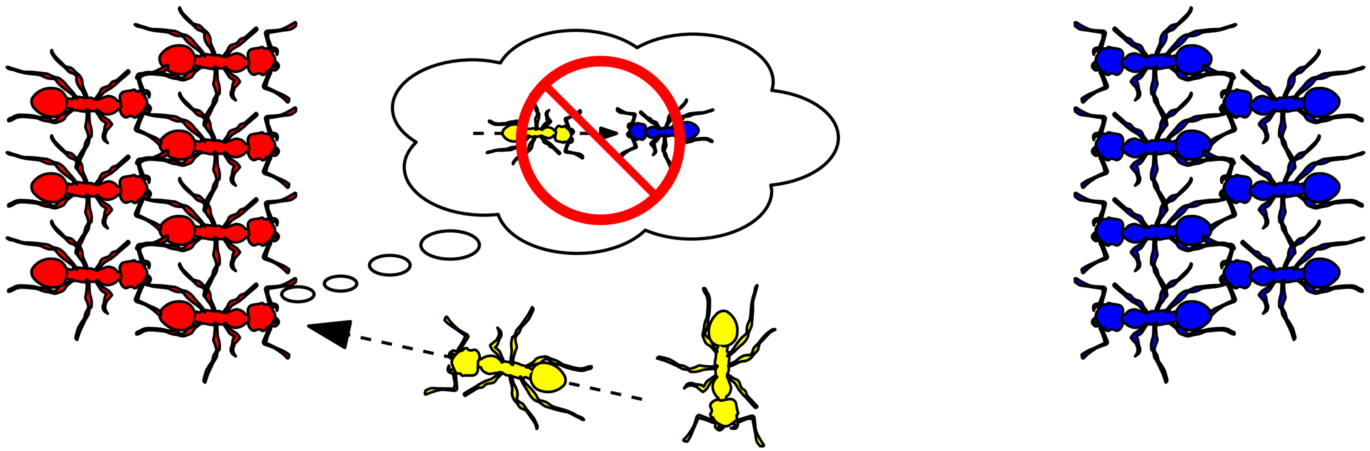

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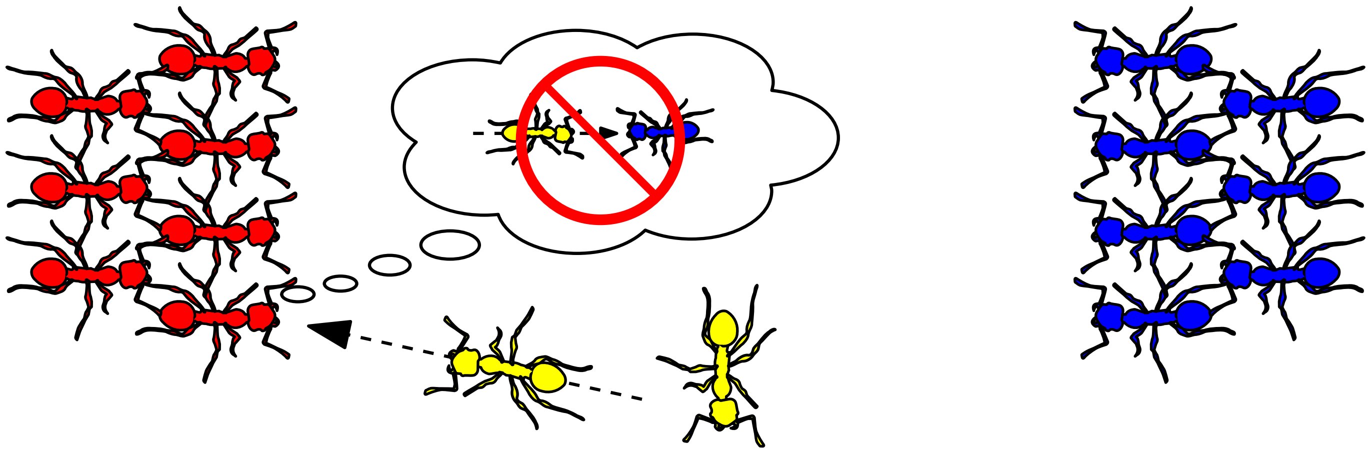


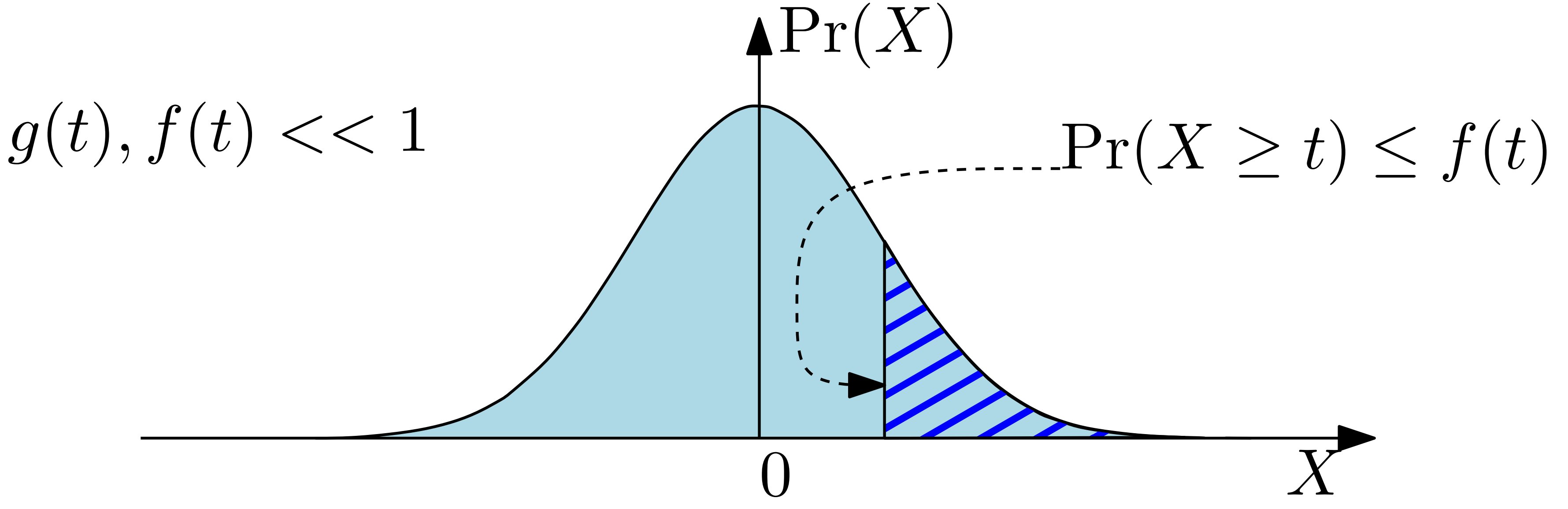
- Multivariate Asymptotics

The number k of *states* of an agent changes with the number of agents in the system.

$$k = k(n) \underset{n \to \infty}{\longrightarrow} \infty$$

Mathematical Challenges

- "Small Deviations"



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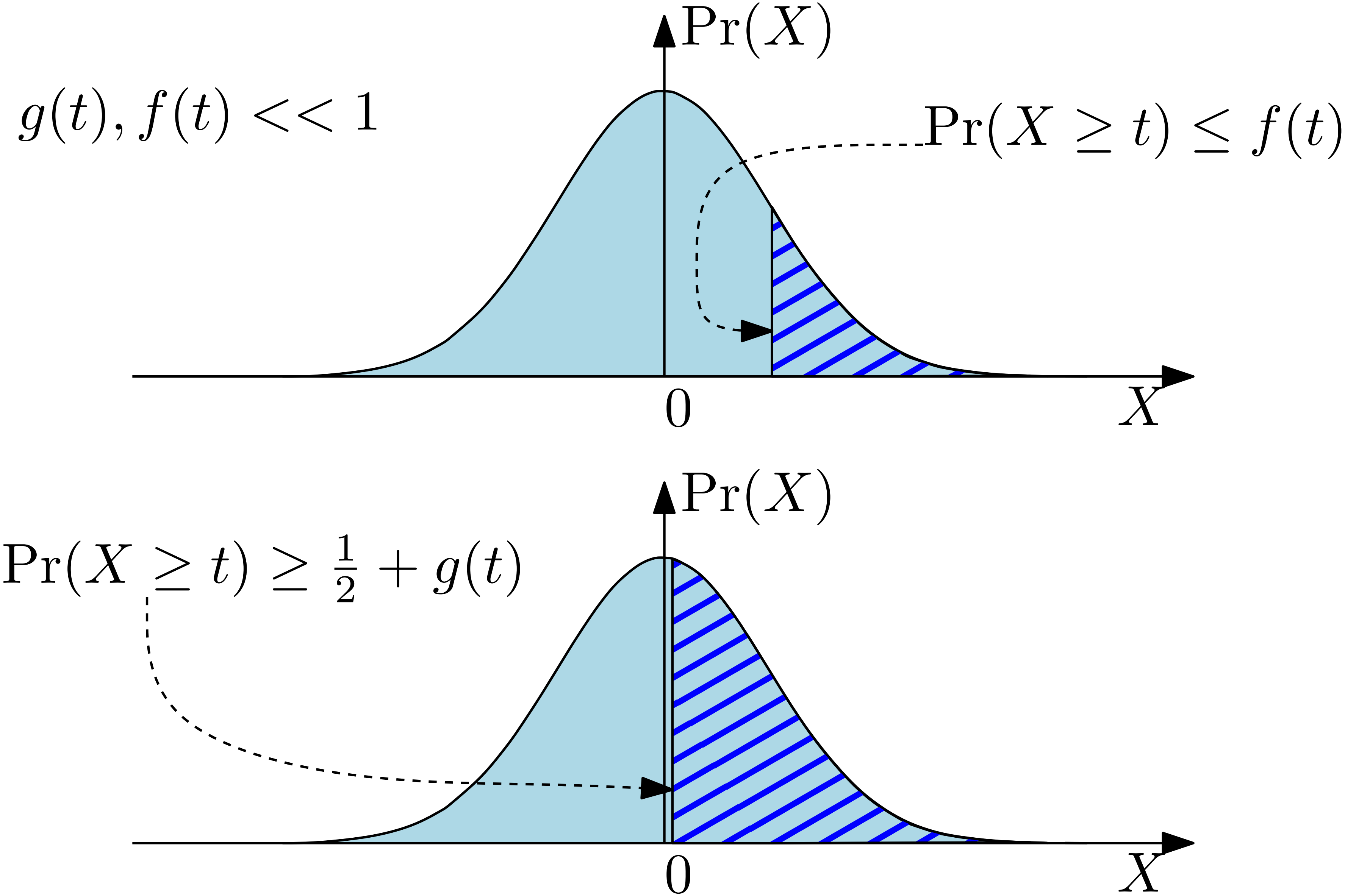
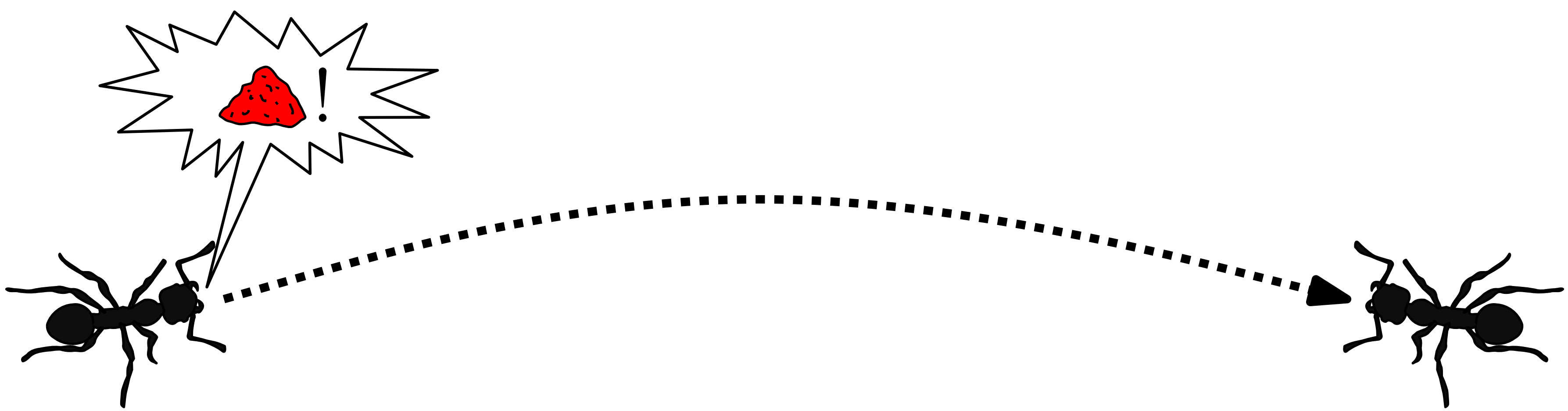


Multivalued Case



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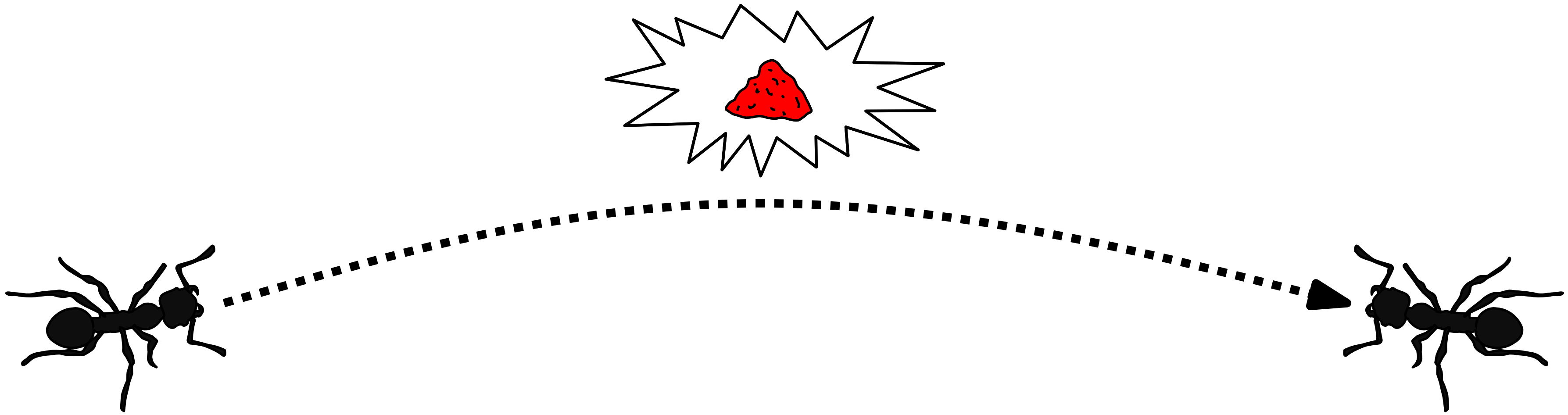

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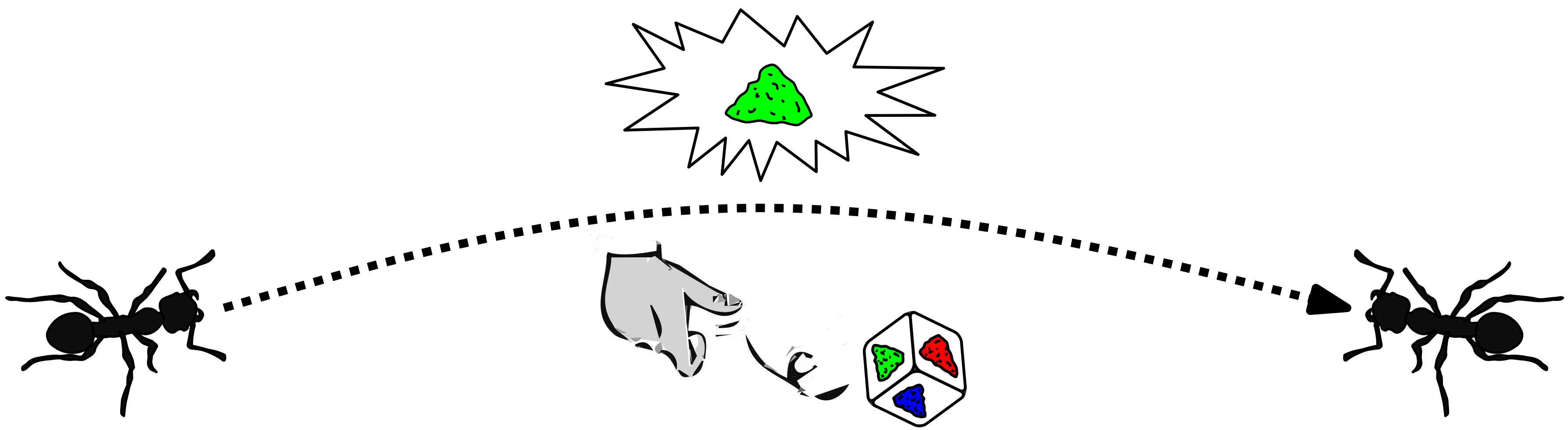

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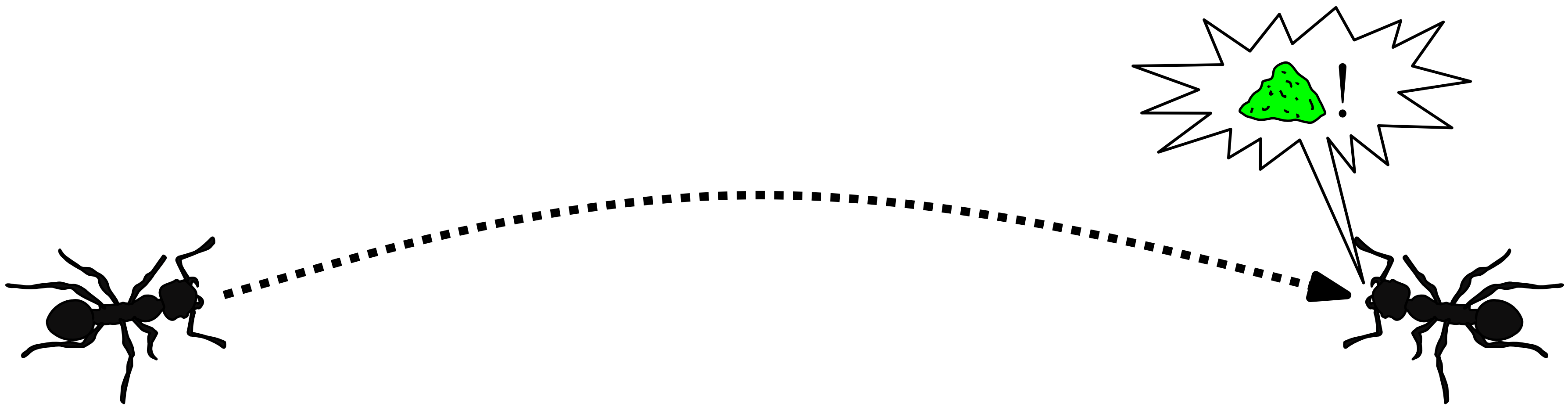
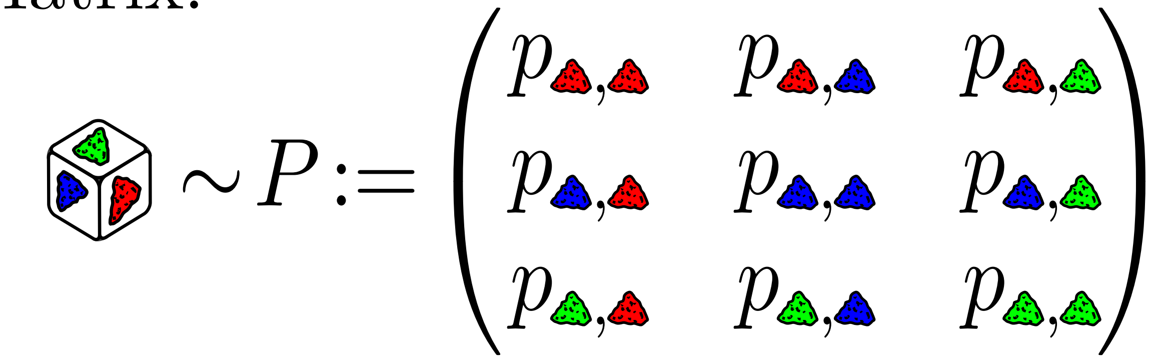
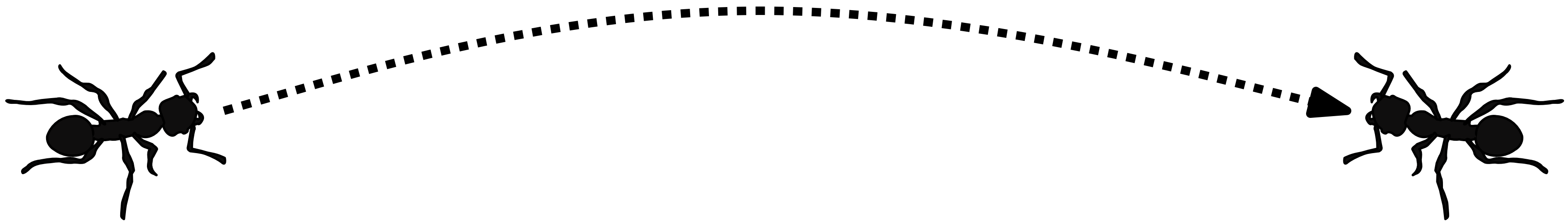


Multivalued Case

Noise Matrix:





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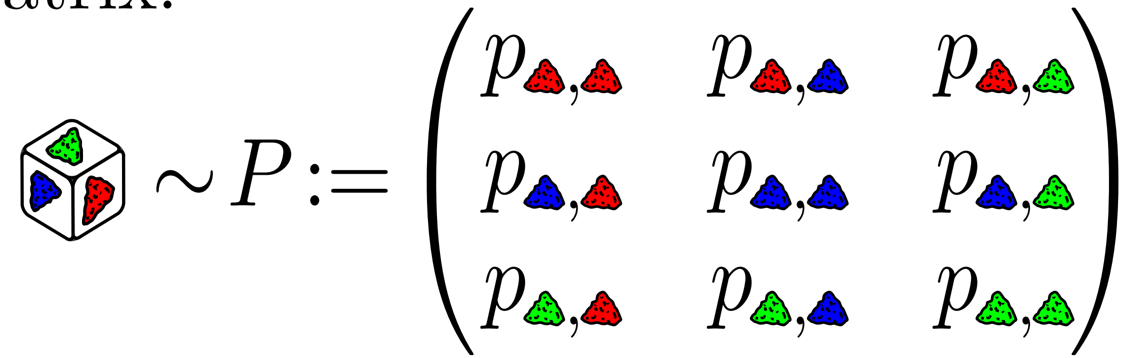
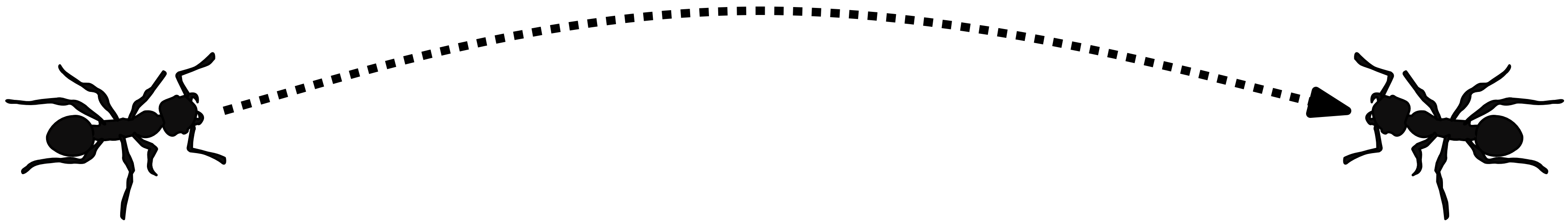
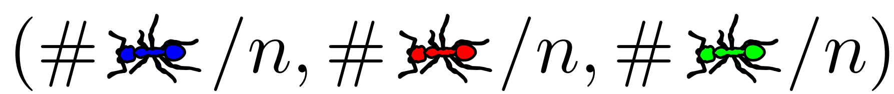
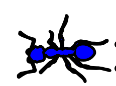
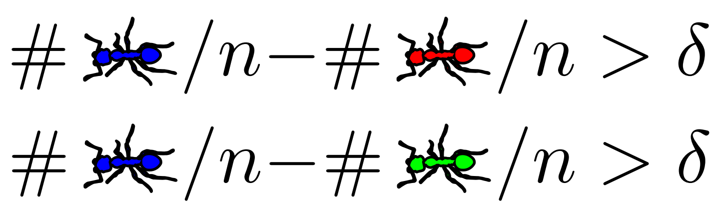




Configuration $\mathbf{c}$:= 

δ-majority-biased configuration w.r.t. :



Main Result

ε-majority-preserving noise matrix:

$(\mathbf{c}P)$ $- (\mathbf{c}P)$ $> \varepsilon\delta$

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Theorem. Let S be the initial set of agents with opinions in $[k]$. Suppose that the noise matrix P is ϵ-majority-preserving and S is $\Omega(\sqrt{\log n/|S|})$-majority-biased with $|S| = \Omega(\frac{\log n}{\epsilon^2})$. Then the rumor spreading and plurality consensus problems can be solved in $O(\frac{\log n}{\epsilon^2})$ rounds w.h.p., with $O(\log \log n + \log \frac{1}{\epsilon})$ memory per node.

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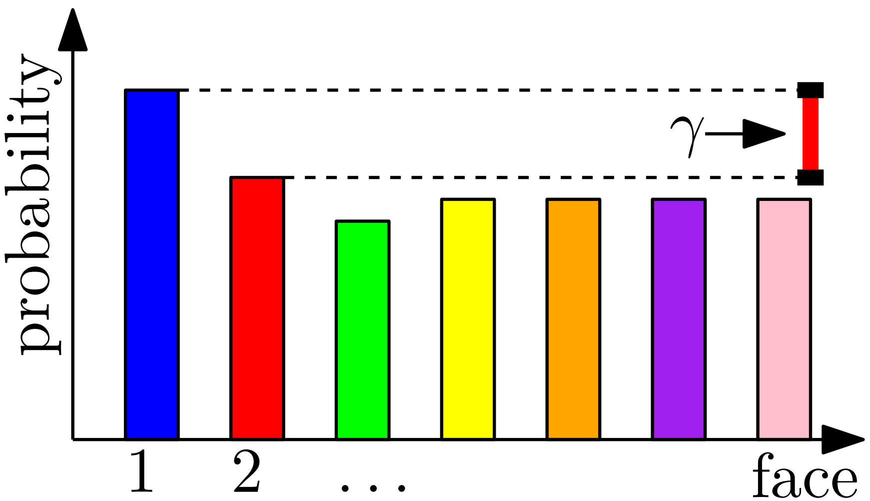
$$P = \begin{pmatrix} 1/2+\varepsilon & 1/2-\varepsilon \\ 1/2-\varepsilon & 1/2+\varepsilon \end{pmatrix} \implies \text{Feinerman et al.}$$

Probability Amplification

A dice with k faces is thrown ℓ times.



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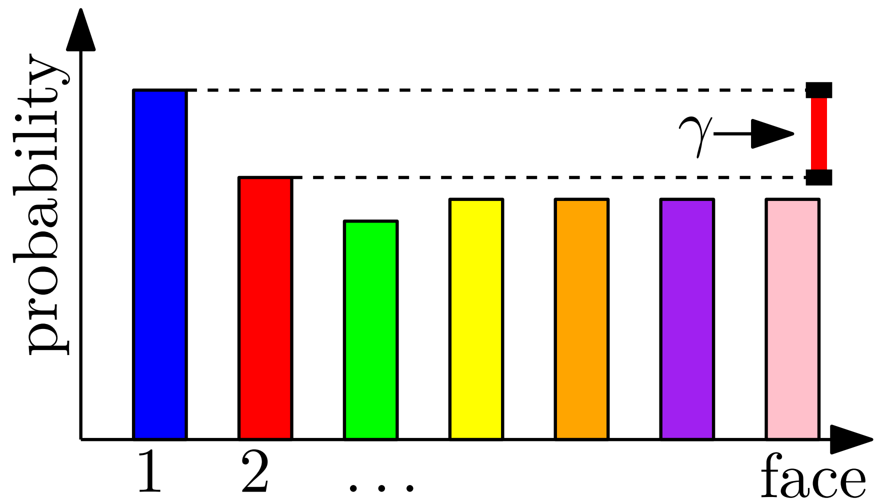

$\mathcal{M}$:= most frequent face in the ℓ throws (breaking ties at random).

For any $j \neq 1$

$$\Pr(\mathcal{M} = 1) - \Pr(\mathcal{M} = j) \geq \text{const} \cdot \sqrt{\ell}\,\gamma(1-\gamma^2)^{\frac{\ell-1}{2}}$$

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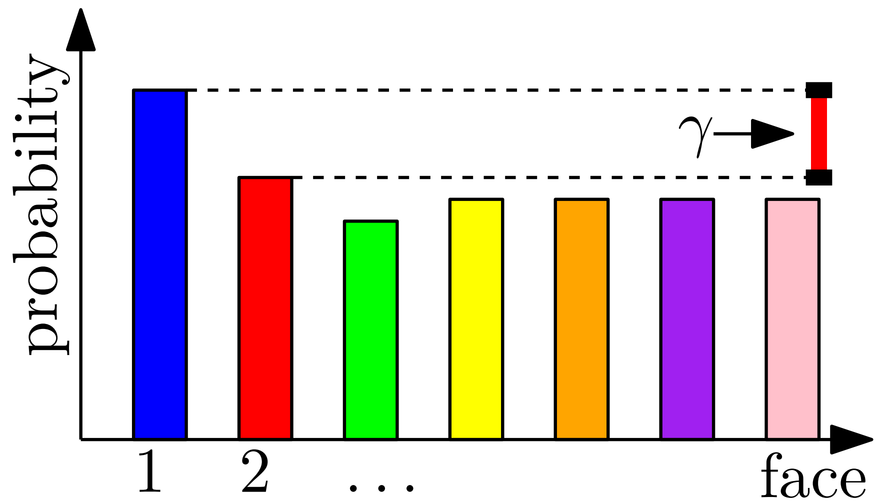
A dice with k faces is thrown ℓ times.



$\mathcal{M}$:= most frequent face in the ℓ throws (breaking ties at random).

For any $j \neq 1$

$$\Pr(\mathcal{M}=1) - \Pr(\mathcal{M}=j) \geq \text{const} \cdot \sqrt{\ell}\,\gamma(1-\gamma^2)^{\frac{\ell-1}{2}}$$

open problem: const $\approx e^{-\Theta(k)}$

Binomial vs Beta

Given $p \in (0,1)$ and $0 \leq j \leq \ell$ it holds

$$\begin{aligned}\Pr\left(Bin(n,p) \leq j\right) &= \sum_{j<i\leq \ell} \binom{\ell}{i} p^i \left(1-p\right)^{\ell-i} \\ &= \binom{\ell}{j+1} (j+1) \int_0^p z^j \left(1-z\right)^{\ell-j-1} dz \\ &= \Pr\left(Beta(n-k,k+1) < 1-p\right).\end{aligned}$$

Binomial vs Beta

Given $p \in (0,1)$ and $0 \le j \le \ell$ it holds

$$\Pr\left(Bin(n,p) \le j\right) = \sum_{j < i \le \ell} \binom{\ell}{i} p^i \left(1-p\right)^{\ell - i}$$

$$= \binom{\ell}{j+1} (j+1) \int_0^p z^j \left(1-z\right)^{\ell-j-1} dz$$

$$= \Pr\left(Beta(n-k, k+1) < 1-p\right).$$

Multinomial vs Dirichlet?

Thank You!